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## Sea Ice Koopman Mode Analysis

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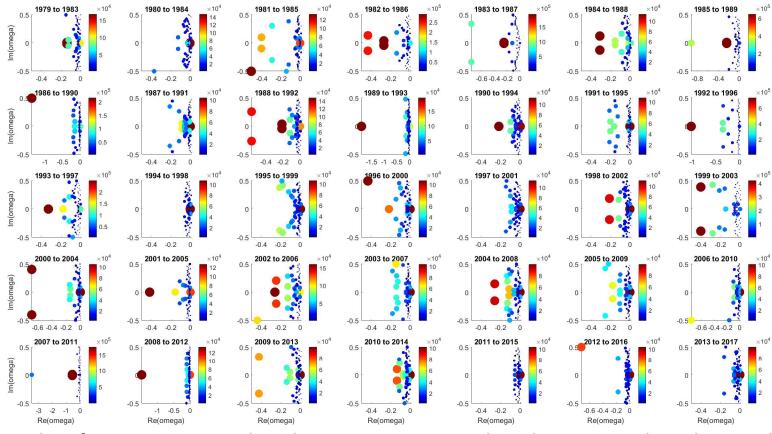
#### Overview



- The program goal is to apply Koopman Mode Analysis (KMA) to sea ice data sets to study their temporal and geographic behavior over multi-year time scales.
- The data sets examined to date consist of sea ice concentration for the northern and southern hemispheres from the NSIDC Sea Ice Index, and sea ice thickness for the northern hemisphere from the CryoSat-2 satellite.
- Results of interest include
  - 1. Koopman modes showing the decrease in the mean and annual variation in sea ice concentration in both hemispheres since 1979, particularly in the regions of West Antarctica and north of Alaska and eastern Siberia.
  - 2. Local variations in sea ice thickness north of Svalbard.

#### Sea ice concentration, northern hemisphere Koopman eigenvalues 5 year windows

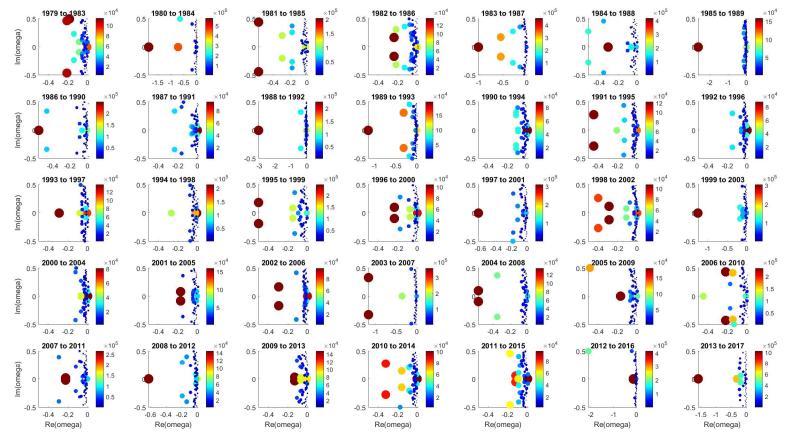




Koopman eigenvalues for 5 year time windows beginning in 1979. Each circle corresponds to the complex frequency of a Koopman mode, where the size and color of the circle are both representations of the L2 norm of the corresponding mode, the position along the horizontal axis shows the growth or decay constant of the mode (e.g. the more negative the value the faster the mode decays), and the position along the vertical axis shows the oscillatory frequency of the mode. The time scale is in months, so the two circles generally visible at approximately +/-0.08 on the vertical axis are the expected annual variation of the sea ice concentration. The novel result apparent is the mostly consistent presence of a large norm mode near the origin beginning in the 1990-1994 window. This slowly decaying or growing mode suggests a change in sea ice dynamics beginning in the 1990's.

#### Sea ice concentration, southern hemisphere Koopman eigenvalues 5 year windows

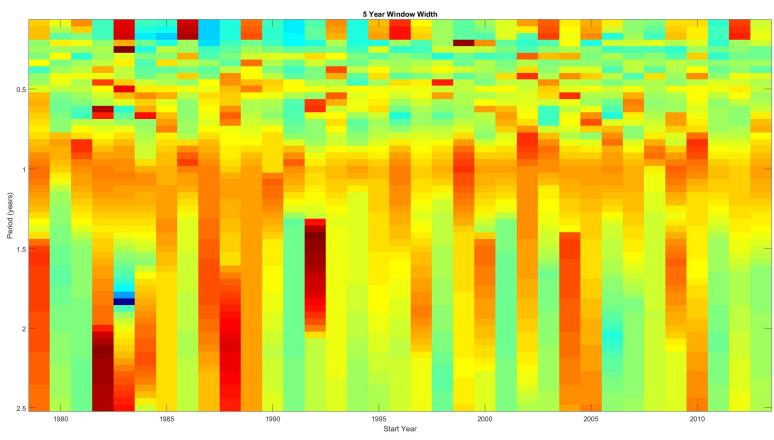




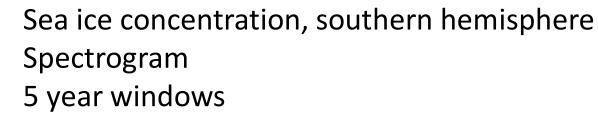
Koopman eigenvalues for 5 year time windows beginning in 1979. Each circle corresponds to the eigenvalue of a Koopman mode, where the size and color of the circle are both representations of the L2 norm of the corresponding mode, the position along the horizontal axis shows the growth or decay constant of the mode (e.g. the more negative the value the faster the mode decays), and the position along the vertical axis shows the oscillatory frequency of the mode. The time scale is in months, so the two circles generally visible at approximately +/-0.08 on the vertical axis are the expected annual variation of the sea ice concentration. The novel result apparent is the mostly consistent presence of a large norm mode near the origin beginning in the 1990's.

#### Sea ice concentration, northern hemisphere Spectrogram 5 year windows

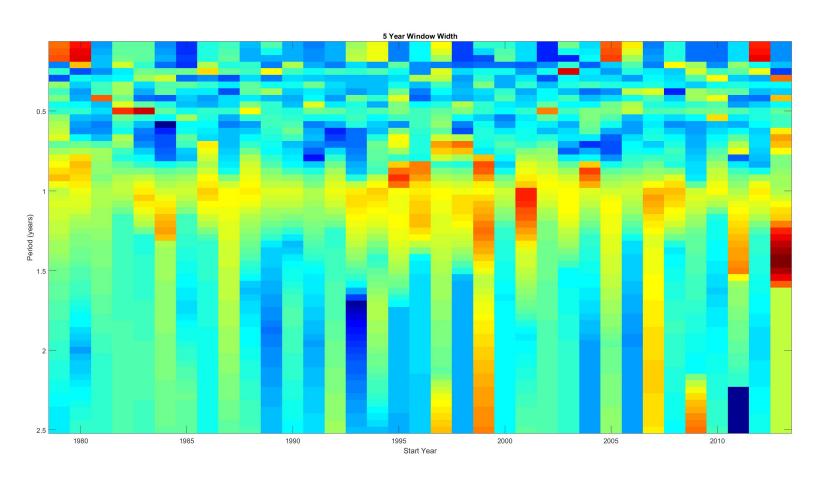




Spectrogram of the Koopman mode spectra for 5 year time windows beginning in 1979. The period (in years) is shown here instead of frequency, because the multiyear dynamics are of greater interest than month-scale behavior. The expected annual cyclical behavior is clearly visible in each time window, and harmonics at two years are also apparent in many cases.







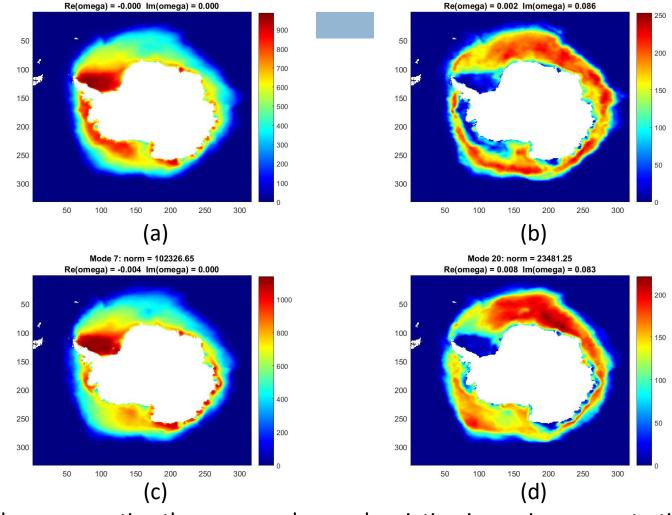
Koopman spectrogram showing period (in years) of eigenvalues. The consistent peak at one year represents the annual variation in sea ice concentration.



Sea ice concentration, southern hemisphere Koopman modes: Mean and annual variation

Mode 4: norm = 90661.44

5 year windows

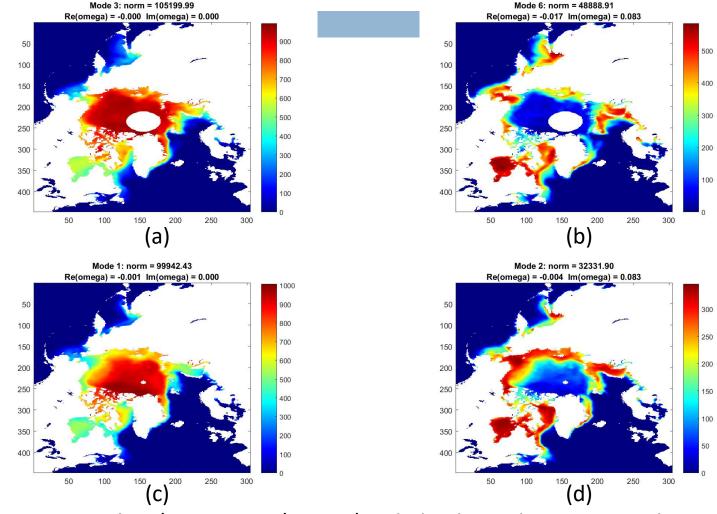


Mode 13: norm = 26489.07

Koopman modes representing the mean and annual variation in sea ice concentration over five year windows for the southern hemisphere. (a) Mean coverage, 1979-1983 period, (b) annual variation, 1979-1983 period, (c) mean coverage, 2013-2017 period, (d) annual variation, 2013-2017 period. A general decrease in sea ice coverage is apparent, particularly near West Antarctica.



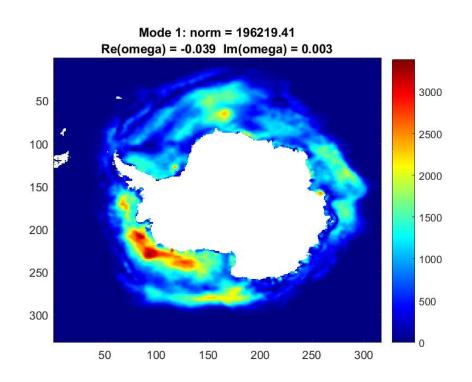
#### Sea ice concentration, northern hemisphere Koopman Modes: Mean and annual variation 5 year windows



Koopman modes representing the mean and annual variation in sea ice concentration over five year windows for the northern hemisphere. (a) Mean coverage, 1979-1983 period, (b) annual variation, 1979-1983 period, (c) mean coverage, 2013-2017 period, (d) annual variation, 2013-2017 period. A general decrease in sea ice coverage is apparent, particularly in the Beaufort and Kara seas.



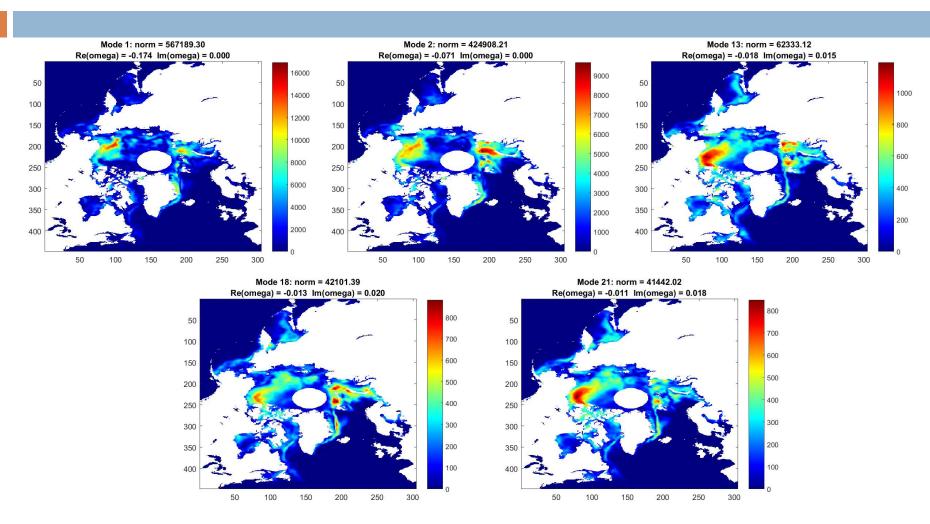
Sea ice concentration, southern hemisphere Koopman mode: Long term decrease 39 year window



Mode from entire 39 year data set showing long term decrease in sea ice off West Antarctica.



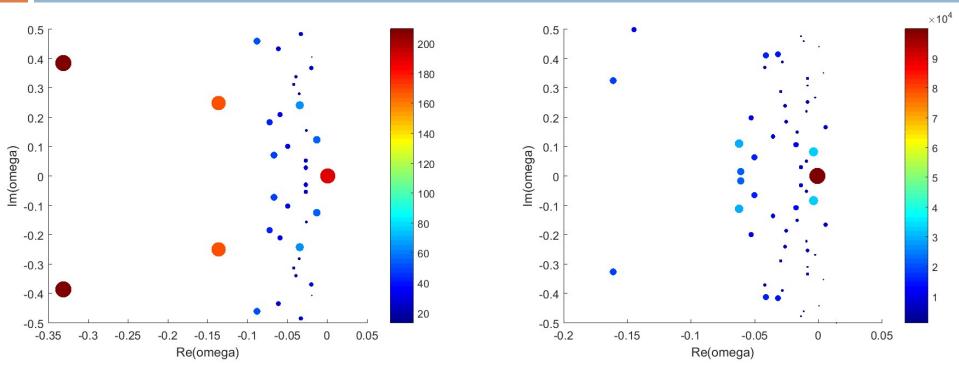
### Sea ice concentration, northern hemisphere Koopman modes: Long term decrease 39 year window



Modes from entire 39 year data set for the northern hemisphere, showing long term decrease in sea ice north of Alaska and European Russia.



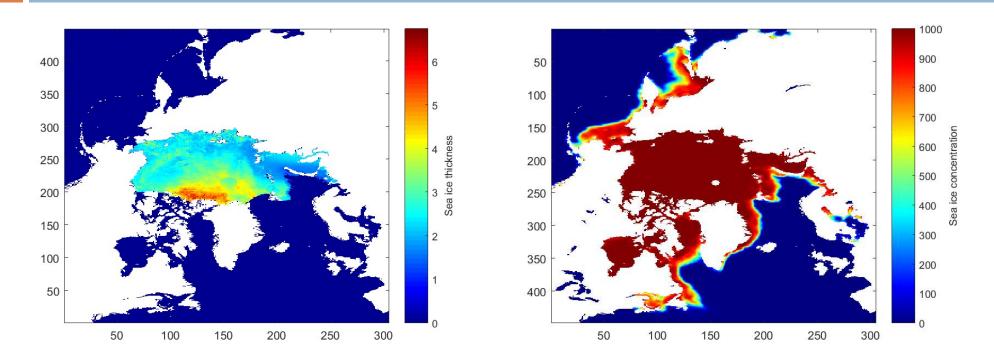
### Sea ice thickness and concentration, northern hemisphere Koopman eigenvalues 5 year windows



Comparison of Koopman eigenvalues for sea ice thickness and concentration over a five year window of 2013 to 2017 for the northern hemisphere. Left: Thickness eigenvalues, Right: concentration eigenvalues. Both cases show conjugate pairs of modes corresponding to annual variation (at +/-0.125 on the vertical axis for the thickness data and at +/-0.083 for the concentration data) and biannual variation (at +/-0.250 and +/-0.167 on the vertical axes for thickness and concentration data, respectively) as well as a generally similar distribution of other modes.

#### Sea ice thickness and concentration Maximum extent and values



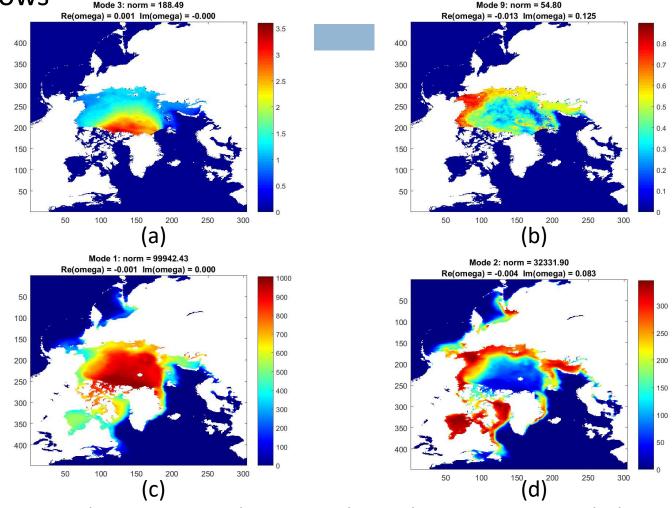


Maximum values obtained for each pixel in the thickness and concentration data sets. The thickness data is only available over a restricted region in and around the Arctic Ocean, while the concentration data is available over a wider region. The thickness data is also seen to have more structure visible because the maximum thickness values are not bounded, whereas concentration cannot exceed 100% (a pixel value of 1000).



Sea ice thickness, northern hemisphere Koopman modes: Mean and annual variation





Comparison of Koopman modes representing the mean and annual variation in sea ice thickness and concentration over a five year window of 2013 to 2017 for the northern hemisphere. (a) Thickness mean coverage, (b) thickness annual variation, (c) concentration mean coverage, (d) concentration annual variation. The same geographic patterns are apparent in the mean and annual variations, in that the mean thickness and concentration are largest to the north of Greenland, and the annual variation is largest north of Alaska and eastern Siberia. An interesting difference in the annual variation modes is the presence of localized spots of annually varying thickness north of Svalbard.